MODEL OF LINEAR DISPLACEMENT OF WATER BY OIL FROM A POROUS MEDIUM SPECIMEN

Yu. A. Afinogenov and V. I. Pen'kovskii

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 1, pp. 144–147, 1969

As is known [1], the system of equations describing two-phase motion of liquids with different physicochemical characteristics in a porous medium is rather complex and can be integrated only by numerical methods. This is a result both of the complexity of the system and of the fact that the functional dependences of the relative phase permeabilities and capillary pressure on the saturation which appear in these equations are often given in the form of experimental curves.

Assumptions of various sorts are used in order to obtain analytic solutions of the two phase-motion problem. Here we assume that the real porous medium specimen can be replaced by a system of parallel capillaries [2], whose radii r in unit cross section of the specimen are distributed in accordance with the log-normal law with density

$$f(r) = n_0 \exp\left\{-\frac{\ln^2 r/r_0}{2\sigma^2}\right\}, \tag{1}$$

where n_0, r_0, σ are parameters. Several experimental investigations support the introduction of such a distribution (see, for example [3]). It is natural to select the constants n_0, r_0, and σ so that the basic filtration characteristics of the real medium and the model coincide.

We first examine the motion of a homogeneous liquid with viscosity μ under the influence of the pressure gradient $dp/dx = \Delta p/l = \text{const}$. Here Δp is the pressure difference across the ends of the speciment of length l; x is the coordinate axis along the specimen, which is positioned horizontally.

The number dN of capillaries whose radii lie in the interval $(r, r + dr)$ is $dN = f(r)dr$, so that the total number of capillaries per unit section will be

$$N = \int_0^\infty f(r)\,dr = \sqrt{2\pi}\, n_0 r_0 \sigma \exp\left\{\frac{\sigma^2}{2}\right\}. \tag{2}$$

The dimension of the number N is cm^{-2}. It is obvious that the minimum r_* and maximum r^* radii of the capillary are found from the relations

$$\int_0^{r_*} f(r)\,dr = 1, \qquad \int_{r^*}^\infty f(r)\,dr = 1.$$

The effective porosity m of the model specimen is written as

$$m = \pi \int_0^\infty r^2 f(r)\,dr = \pi N r_0^2 \exp\{4\sigma^2\}. \tag{3}$$

From the Hagen–Poiseuille formula we have the flow rate q through a capillary of radius r that

$$q = -\frac{\pi r^4}{8\mu}\frac{dp}{dx}.$$

Consequently the total flow rate Q through a unit section of the model is

$$Q = -\frac{\pi}{8\mu}\frac{dp}{dx}\int_0^\infty r^4 f(r)\,dr.$$

Comparing this with the Darcy law for the real porous medium with permeability k

$$Q = -\frac{k}{\mu}\frac{dp}{dx}.$$

we obtain

$$k = \frac{\pi}{8} \int_0^\infty r^4 f(r)\, dr = \frac{\pi}{8} r_0^4 N \exp\{12\sigma^2\}. \tag{4}$$

From (3) and (4) we find

$$\sigma^2 = \frac{1}{8} \ln \frac{1}{a} \qquad \left(a = \frac{m r_0^2}{8k}\right) \tag{5}$$

To obtain the third relation connecting the parameters of the function $f(r)$, we can require that one of the important characteristics of the two-phase motion process — the residual water saturation s_0 — be the same for the real medium and the model.

It is known [2] that in the case of oil injection into a specimen previously completely saturated with water the water displacement begins only for pressures above some definite minimum "displacement pressure" Δp_* and that there always remains the so-called residual water saturation, which is practically independent of the magnitude of the pressure differential created across the end of the specimen. This phenomenon is explained by the hydrophilic nature of the medium. In accordance with the Laplace formula, the capillary pressure preventing oil penetration in capillaries of radius r will be

$$p_c = c / r \quad (c = 2\sigma_{12} \cos \theta) \tag{6}$$

Here σ_{12} is the interphase tension force at the water-oil boundary and θ is the contact wetting angle.

As a result of the hydrophilic nature of the medium, a water film remains on the walls of the pores (capillaries). We henceforth assume that the film thickness δ is constant and independent of the capillary radius and that

$$\Delta p > c / r^* = \Delta p_*.$$

Let μ_1, μ_2 be the oil and water viscosities, respectively.

Using the formula obtained by Kotyakhov [4], we find the distance x(r,t) traveled by the water-oil contact surface from the point x = 0 during the time t in the capillary of radius r (Fig. 1, where 1 is the oil region, 2 is the water region):

$$x(r, t) = \frac{\mu_2 l - \sqrt{(\mu_2 l)^2 - t\Delta\mu\, (\Delta p - p_c)(r - \delta)^2 / 4}}{\Delta\mu}. \tag{7}$$

$$(\Delta\mu = \mu_2 - \mu_1)$$

We see from (7) that x(r,t) is an ascending function of the variable r and, consequently, the oil will fill primarily the capillaries with large radii. The leading edge of the water-oil contact surface propagation in the model speciment is obviously

$$x^*(t) = x(r^*, t).$$

The first oil portions travel through the specimen in the time t_*, given by the formula

$$t_* = \frac{4l^2 (\mu_2 + \mu_1)}{(\Delta p - \Delta p_*)(r^* - \delta)^2}. \tag{8}$$

Let us examine the specimen section $x = x(r,t)$, given by (7) and located between the points with the abscissas: $x = x_*(t)$ and $x = x^*(t)$.

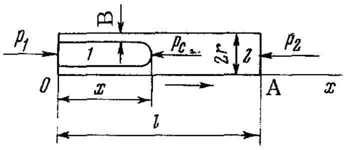

Fig. 1

At the time t all the capillaries at this section whose radii are greater than r will be filled with oil; the capillaries with smaller radii will be saturated with water. Thus, the oil saturation s at this section is defined in the form

$$s(r) = \frac{1}{m} \int_{r}^{\infty} \pi (r - \delta)^2 f(r)\, dr. \tag{9}$$

Taking (9) into account, we determine the magnitude of the residual water saturation s_0, which is dependent of the pressure differential (in view of the adopted assumption that the thickness of the water film wetting the capillary walls is constant), in the form

$$s_0 = 1 - s(\delta). \tag{10}$$

Thus, if the porosity, permeability, and residual water saturation of the real medium are given, then (3), (4), and (10) can be used to construct a capillary model having the same characteristics as the real medium, and the process of water displacement by oil is described by simple analytic expressions.

Excluding from (9) and (7) the parameter r, which, generally speaking, runs through the half-interval $[\delta, \infty]$, we find the oil saturation profiles at any time t, i.e., a relation of the form $s = s(x,t)$.

Assuming that filtration through the specimen can take place for any given pressure gradients Δp_i, just as under natural conditions of the oil stratum disposition, and using a formula of the form (6), for $r_i^* = c/\Delta p_i$ we can find the time for displacement of the free water from the specimen from (8).

To find the oil phase permeability k_1^0 we use the so-called weighted average permeability for the nonwetting phase [5,6]

$$k_1{}^{\circ} = s(r^*) \int_{r^*}^{\infty} (r - \delta)^4 f(r)\, dr \left(\int_{0}^{\infty} r^4 f(r)\, dr \right)^{-1}. \tag{11}$$

We obtain similarly the wetting phase (water, K_2°) permeability

$$k_2{}^{\circ} = [1 - s(r^*)] \left(\int_{0}^{r^*} r^4 f\, dr + \int_{r^*}^{\infty} [r^4 - (r - \delta)^4]\, f\, dr \right) \left(\int_{0}^{\infty} r^4 f\, dr \right)^{-1}. \tag{12}$$

For a pressure differential $\Delta p = \text{const}$, displacement of water by oil will take place in all capillaries whose radii r are larger than r^*; stationary water will be present in the other capillaries. In this case the oil will travel selectively along the capillaries of largest radius, forming a water-oil contact front which varies with time t and approaches the opposite end of the model specimen. At the time t_1 the opposite end is reached by the front point of the water-oil contact front surface corresponding to the maximum capillary radius $r^{\max}$, then at the time t_2 the opposite end is reached by the lagging point of the water-oil contact front surface corresponding to the minimum radius $r^{\min}$ of the capillary through which oil can still travel for the Δp assumed; the times t_1 and t_2 are found from (8) for $r^* = r^{\max}$ and $r^* = r^{\min}$, respectively.

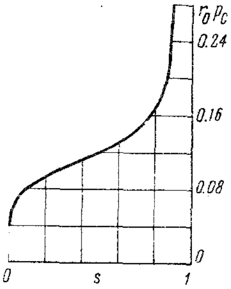

Fig. 2

In the course of the time (t_1, t_2) the oil flow rate through the opposite end of the specimen will be variable. Kotyakhov [4] obtained the formula for the flow rate of liquids through capillaries

$$b(r, t) = \frac{\pi (r - \delta)^4 (\Delta p - p_c)}{8 \sqrt{(\mu_2 l)^2 - (r - \delta)^2 \Delta\mu (\Delta p - p_c)\, t / 4}}. \tag{13}$$

Substituting into (13) expression (8) for t with $r^* \in [r^{\min}, r^{\max}]$, we obtain the flow rate $b_1(r^*)$ through the capillaries at the opposite end of the specimen. Forming the product $b_1(r) f \Delta r$ and summing in the limits from $r^{\min}$ to $r^{\max}$, (where $r^{\max} \sim \infty$), we obtain the formula for the oil discharge during the time interval (t_1, t_2):

$$Q = \int_{r^{\min}}^{\infty} b_1(r) f \, dr.$$

However, if we take $r^* = r^{\min}$ as a function of the time t from (8), we obtain the oil discharge through the opposite end of the specimen in the half-interval $[t_1, t]$, where t_1 corresponds to $r^{\max} \sim \infty$:

$$Q(t) = -\int_{t}^{t_1} b_1(\tau) f(\tau) J(\tau) \, d\tau$$

$$(dr(\tau) = J(\tau) d\tau).$$

Here τ is the variable of integration.

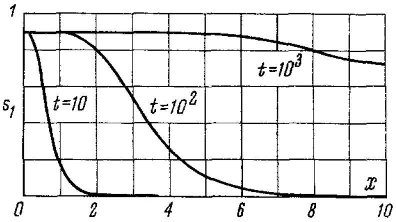

Fig. 3

Generally speaking, at the opposite end of the specimen both the residual water saturation and the permeability will be variable in the interval (t_1, t_2).

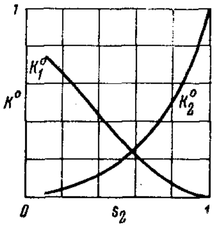

Fig. 4

In the case of oil flow along the capillaries $\{r\} \in [r^{\min}, r^{\max}]$, we can find how the oil saturation varies at the sections of the water-oil contact front for $t = t_1$:

$$s(x, t_1) = \int_{r(x, t_1)}^{\infty} (r - \delta)^2 f \, dr \left(\int_{0}^{\infty} r^2 f \, dr \right)^{-1}.$$

In the unsteady filtration regime, when the pressure differential is a function of time, i.e., $\Delta p = \Delta p(t)$, the filtration characteristics s and k^0 will also be functions of time.

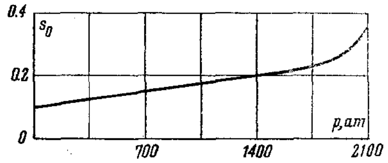

Fig. 5

Knowing, for example experimentally or theoretically, how the porosity and permeability of rock specimens vary as a function of the effective rock pressure and temperature, and also having one other value (for example, the variation of r_0, the parameter $f(r)$ as a function of the effective rock pressure and temperature),— we can express the parameters of the distribution $f(r)$ in terms of the effective rock pressure and temperature, and using these parameters for the model specimen described above we can find s_0 and k^0 as a function of the effective rock pressure and temperature.

Figures 2, 3, and 4 show curves of the capillary pressure, oil saturation profiles, and phase permeabilities, respectively, calculated using (6), (7), (9), (11), (12) for $k_0 = 0.4065$ darcy, $m_0 = 0.182$, $r_0 = 3 \cdot 10^{-4}$ cm, $\sigma = 0.3$,

$a = 0.5$, $\delta = 0.2 \cdot 10^{-4}$ cm, $\mu_2 = 0.01$ cp, $\mu_1 = 0.11$ cp, $l = 10$ cm, $\Delta p = 5075 \cdot 10^2$ dynes/cm^2 for $t = 10, 10^2, 10^3$ sec.

Figure 5 presents the relation $s_0 = s_0 (p_1)$, where p_1 is the effective rock pressure. Analysis of core material at the lithology laboratory of the Siberian Scientific Research Institute of Geology, Geophysics, and Mineral Raw Materials has shown that the following empirical relations hold:

$$k = \frac{210}{523 + p_1} 10^{-8} \text{ cm}^2, \quad r_0 = \frac{18}{600 + p_1} 10^{-4} \text{ cm}^2,$$

$$m = [(1 / m_0 - 1) \exp (0.015 p_1^{0.5140}) + 1]^{-1} .$$

REFERENCES

1. I. A. Charnyi, Underground Hydrogasdynamics [in Russian], Gostoptekhizdat, Moscow, 1963.
2. A. E. Scheidegger, The Physics of Flow Through Porous Media [Russian translation], Gostoptekhizadt, Moscow, 1960.
3. A. F. Bogomolova and N. A. Orlova, "Quantitative characteristics of pore space structure," PMTF, no. 4, 1961.
4. F. I. Kotyakhov, "Motion of the meniscus of two immiscible liquids in capillaries," Problems in the Development and Exploitation of Oil Deposits [in Russian], Tr. Vses. n.-i. in-ta, no. 2, Gostoptekhizdat, Moscow, 1952.
5. E. M. Minskii, "Elements of the statistical study of gas filtration motions," Development of Gas Deposits, Gas and Oil [in Russian], Transport and Economics Tr. Vses. n.-i. in-ta prirodnykh gazov, no. 2, 1958.
6. M. I. Vainer, "Statistical similarity criteria for liquid filtration in a homogeneous porous medium," Izv. AN SSSR, OTN, Mekhanika i mashinostroenie, no. 5, 1963.

13 August 1968

Novosibirsk